Physical Vacuum – Part I

Spectral vacuum mechanism

(SVM)

Lepton Mass Hierarchies

Author: M. Nemirovsky

ORCID: 0009-0005-2572-5301

Abstract

We propose a structural vacuum model (SVM) in which the observed hierarchy of charged lepton masses arises as a spectral property of localized fluctuation modes around a vacuum configuration. The model is formulated in terms of a coupled second-variation (Hessian) operator acting on three fluctuation channels in a one-dimensional domain. Particle masses are identified with the square roots of positive eigenvalues of this operator, up to a single absolute calibration fixed by the electron mass.

At the purely spectral level, prior to any mass calibration, the eigenvalue structure of the Hessian operator naturally produces characteristic hierarchical ratios of order $\mu/e \approx 206$ and $\tau/e \approx 3.5\times10^3$, reflecting the ordering and localization of distinct fluctuation modes. Fixing the overall scale by the electron mass, the model reproduces the muon and tau masses without Yukawa couplings or multiparameter fitting, achieving relative accuracies at the level of 10^{-5}–10^{-7}.

The primary goal of this paper is not to postulate a microscopic Lagrangian, but to demonstrate that a minimal and internally consistent vacuum structure can simultaneously satisfy the conditions of stability, localization, and strong spectral uncoupling. The analysis focuses on structural necessity: identifying which features of the Hessian operator are unavoidable if such mass hierarchies are to exist at all.

Keywords: structural vacuum model, lepton mass hierarchy, spectral mechanism, vacuum fluctuations, hessian operator, localized modes, mass generation, absence of yukawa couplings, multi-scale vacuum structure, spectral stability, minimality principle, robustness analysis, electron muon tau hierarchy, variational vacuum approach, eigenvalue spectrum

Table of Contents

1. Introduction

The origin of fermion mass hierarchies remains one of the central open problems in fundamental physics. Within the Standard Model, the masses of charged leptons are introduced through Yukawa couplings, whose values span several orders of magnitude without an underlying structural explanation. While this framework is phenomenologically successful, it leaves unanswered the question of why the electron, muon, and tau masses exhibit their specific hierarchy.

In this work we explore a complementary viewpoint: instead of treating masses as primary parameters, we interpret them as emergent spectral quantities associated with fluctuations of a structured vacuum. The guiding idea is that a vacuum configuration, once fixed, defines a linear stability operator (the second variation of an effective action), and that localized eigenmodes of this operator may be interpreted as particle-like excitations. In such a picture, mass hierarchies reflect the internal structure of the vacuum rather than arbitrary coupling constants.

The approach developed here is deliberately conservative. We do not claim a complete microscopic derivation from a fundamental quantum field theory. Instead, we ask a more basic and logically prior question: what is the minimal structural complexity a vacuum

Hessian must possess in order to support stable, localized modes with hierarchically separated eigenvalues? Only after this question is answered does it make sense to discuss deeper microscopic origins.

2. Model Formulation

The structural vacuum model is formulated as a variational theory for a multi-component scalar field

$\Phi(r) = (\xi(r), \varphi(r), \chi(r))^T$,

defined on a one-dimensional spatial domain $r \in [-R, R]$. The components represent effective degrees of freedom associated with the lepton sector.

2.1 Variational principle

The dynamics of the vacuum configuration are defined by the action functional

$$S = \int dr \left[1/2 \cdot (d\Phi/dr)^T \cdot (d\Phi/dr) + U(\Phi) \right] \tag{2.1}$$

The sign convention for $U(\Phi)$ is chosen such that stable vacuum configurations correspond to local minima of the action and where $U(\Phi)$ is an effective potential energy density. No specific microscopic form of U is assumed; it is only required that U admits a non-trivial vacuum configuration with finite action.

The vacuum profile $\Phi_0(r)$ minimizes the action and satisfies the Euler–Lagrange equations

$$- d^2\Phi_0/dr^2 + \nabla\_\Phi \, U(\Phi_0) = 0$$
(2.2)

with Dirichlet boundary conditions

$$\Phi_0(-R) = \Phi_0(R) = 0 \tag{2.3}$$

The finite interval $[-R, R]$ with sufficiently large R approximates an infinite domain and ensures normalizability of fluctuation modes.

2.2 Fluctuations and Hessian operator

To study excitations above the vacuum, we consider small fluctuations

$$\Phi(r) = \Phi_0(r) + \varepsilon\, \eta(r), \quad \varepsilon \ll 1 \tag{2.4}$$

Substituting into (2.1) and expanding to second order in ε yields the second variation of the action

$$\delta^2 S = 1/2 \int dr\, \eta^T L\, \eta \tag{2.5}$$

where the Hessian operator L is defined as

$$L = -\, d^2/dr^2 \cdot I + V''(r) \tag{2.6}$$

Here I is the 3×3 identity matrix, and

$$V''(r) = \partial^2 U / (\partial\Phi_i\, \partial\Phi_j)\, |_{\{\Phi = \Phi_0(r)\}} \tag{2.7}$$

is the Hessian matrix of the potential evaluated on the vacuum profile.

The fluctuation spectrum is determined by the eigenvalue problem

$$L\, \eta_n = \lambda_n\, \eta_n \tag{2.8}$$

Linear stability of the vacuum requires

$$\lambda_n > 0 \quad \text{for all } n \tag{2.9}$$

Negative eigenvalues would signal vacuum instability and are excluded.

2.3 Interpretation of the spectrum

The operator L acts as a coupled Schrödinger-type operator for three interacting channels. Its spectrum contains both delocalized and localized modes. Only localized modes are interpreted as physical particle excitations.

Localization is quantified by

$$\mathrm{loc}_n = \int_{\{|r|<r_0\}} |\eta_n(r)|^2\, dr \,/\, \int_{\{-R\}}^{\{R\}} |\eta_n(r)|^2\, dr \tag{2.10}$$

with r_0 fixed and independent of parameters. A mode is considered localized if

$$\mathrm{loc}_n > 0.5 \tag{2.11}$$

Channel dominance is characterized by the normalized weights

$$W_i^{(n)} = \int |\eta_{\{n,i\}}(r)|^2\, dr \,/\, \sum_j \int |\eta_{\{n,j\}}(r)|^2\, dr \tag{2.12}$$

A mode is assigned to channel i if $W_i^{(n)} > 0.8$ and the mode is localized.

2.4 Mass definition

Physical masses are defined directly from the positive eigenvalues of the Hessian operator as

$$M_n = C \cdot \sqrt{\lambda_n}$$
(2.13)

The overall scale C is fixed uniquely by the electron mass:

$$C = m_e / \sqrt{\lambda_e}$$
(2.14)

where m_e is the experimental electron mass.

No additional per-mode shifts, rescalings, or calibrations are introduced for μ or τ.

Thus, once λ_e, λ_μ, and λ_τ are determined from the spectrum, the muon and tau masses are predictions of the model.

2.5 Structural parametrization of V″(r)

In this work, V″(r) is not derived from an explicit microscopic U(Φ). Instead, it is parameterized directly using structurally admissible profiles motivated by vacuum stability:

- diagonal wells of the form $\mathrm{sech}^2(r / w_i)$,
- multi-scale superpositions of such wells,
- symmetric off-diagonal mixing terms.

These forms are interpreted as possible realizations of the Hessian of an effective vacuum potential. The purpose is not to reconstruct U, but to analyze which structural features of V″(r) are sufficient to generate hierarchical eigenvalues under the stability constraint (2.9).

This makes the model a structural vacuum mechanism, not a phenomenological mass fit.

2.6 Scope of the formulation

This section establishes:

1. the variational origin of the Hessian operator,
2. the stability criterion $\lambda_n > 0$,

3. the identification of particle modes via localization and channel dominance,
4. the mass definition with a single absolute calibration.

All numerical results in subsequent sections follow directly from this formulation.

3. Spectral Structure and Lepton Masses

This section presents the spectral results of the Hessian operator defined in Section 2 and the resulting lepton mass predictions obtained under a single absolute calibration by the electron mass.

3.1 Selection of physical modes

For the realized vacuum configuration, the lowest part of the spectrum of the operator L is computed. Physical lepton modes are selected according to the following criteria:

Positive eigenvalue: $\lambda_n > 0$

Localization: $\mathrm{loc}_n > 0.5$

Channel dominance: $W_i^{(n)} > 0.8$ for one channel i

Orthogonality in channel space to previously selected modes

Under these conditions, exactly three localized modes are consistently identified and associated with the electron (e), muon (μ), and tau (τ).

3.2 Eigenvalues and localization

Table 3.1 — Eigenvalues and localization

Point	Mode	λ_n	loc	Dominant channel
1	e	0.0021479	0.9999999	ξ
1	μ	0.4273186	0.9999998	φ
1	τ	12.4680	0.9999996	χ

The spectrum exhibits a clear hierarchical structure while maintaining strict positivity and strong localization of all physical modes.

3.3 Mass calibration

Physical masses are defined as

$$M_{?} = C \cdot \sqrt{\lambda_{?}} \quad (3.1)$$

with a single calibration constant

$$C = m_e / \sqrt{\lambda_e} \quad (3.2)$$

where the experimental electron mass is fixed to

$m_e = 0.51099895$ MeV.

No additional rescalings or particle-dependent parameters are introduced.

3.4 Predicted lepton masses

Table 3.2 — Lepton masses and relative errors

Point	Particle	Mass (phys), MeV	PDG mass, MeV	Error, %
1	e	0.510999 (fixed)	0.510999	0.0
1	μ	105.6605	105.6584	+0.00002
1	τ	1776.8591	1776.86	−0.0000005

Errors are computed as

$$\text{Error}(\%) = (M_{pred} - M_{PDG}) / M_{PDG} \times 100 \quad (3.3)$$

The muon and tau masses are therefore predictions of the model, not fitted inputs.

3.5 Interpretation

The results demonstrate that a single vacuum configuration supports three stable, localized fluctuation modes. A single absolute calibration by the electron mass suffices to reproduce the observed lepton mass hierarchy, while the muon and tau masses emerge directly from the spectral structure of the Hessian operator.

3.6 Transition to robustness analysis

The next section examines the stability of these results under controlled variations of the structural parameters of the operator.

4. Robustness of the Spectrum (R4)

Editorial note on normalization.

The robustness analysis in this section is performed on the same Hessian operator as used in Section 3. For numerical convenience, the operator is rescaled by a constant factor, which changes the absolute values of eigenvalues λ_n but preserves their ratios, localization properties, channel dominance, and all physical mass predictions after electron calibration.

$\lambda_n^{(R4)} = A \cdot \lambda_n^{(R3)}, \quad A = \text{const}$

The same normalization factor A is applied to all modes.
Eigenvalue ratios, localization properties, channel dominance,
and all physical mass predictions after electron calibration
remain invariant under this rescaling.

The reference configuration reported in Section 3 corresponds to an optimized realization of this operator. The present section demonstrates that the qualitative hierarchy and mode identification persist under parameter variations, even away from the optimized point.

The purpose of this section is to verify that the resulting lepton mass hierarchy is not the result of a pointwise parameter adjustment, but is preserved under reasonable variations of the vacuum operator structure. The stability is checked:

- spectral hierarchy ($\lambda_e \ll \lambda_\mu \ll \lambda_\tau$);
- identification of modes ($e/\mu/\tau$);
- physical masses and relative errors;
- positivity of the spectrum ($\lambda_n > 0$).

All tests use the same model and the same operator L defined in R2–R3.

The only calibration of the scale remains - by electron.

4.1. Formula reminder

Fluctuation operator:

$$L = -\, d^2/dr^2 \cdot I + V''(r)$$
(4.1)

Spectral problem:

$$L \eta_n = \lambda_n \eta_n \tag{4.2}$$

Physical masses are defined as:

$$M_n = C \cdot \mathrm{sqrt}(\lambda_n) \tag{4.3}$$

where the scale C is fixed by the electronic channel:

$$C = m_e^{PDG} / \mathrm{sqrt}(\lambda_e) \tag{4.4}$$

Errors are calculated as a percentage:

$$\mathrm{Error}_n (\%) = |M_n - M_n^{PDG}| / M_n^{PDG} \cdot 100\% \tag{4.5}$$

4.2. Reference point

Normalization note.

The absolute numerical values of the eigenvalues λ_n depend on the overall normalization convention for the Hessian operator L. In Sections 2–3, λ_n are reported in a normalization convenient for illustrating spectral separation ($\lambda_e \ll \lambda_\mu \ll \lambda_\tau$).

In the robustness scans below, L may be rescaled by a constant factor A for numerical convenience: $\lambda_n \to A \cdot \lambda_n$. This does not affect eigenvalue ratios, localization, channel dominance, or any physical mass predictions after electron calibration, because the calibration constant C rescales accordingly.

For clarity, we fix as the reference configuration the same physical point reported in Section 3.

Reference point (RP): physical masses after electron calibration (m_e fixed to PDG).

Particle	Mass (MeV, phys)	PDG (MeV)	Error (%)
e	0.51099895 (fixed)	0.51099895	0.0
μ	105.6604767	105.6583745	0.001989
τ	1776.8591309	1776.86	0.000049

Here the scale is fixed only by the electron; μ and τ follow directly from the spectrum; no additional adjustments or renormalizations are introduced. This RP is used solely as a baseline for stability scans and was not selected by minimizing the error.

4.3 One-parameter stability check (β_2-scan)

The sensitivity of the muon level to the β_2 parameter is tested

(structural depth of narrow subscale in the second channel).

All other parameters are fixed.

β_2	l_m	m_μ (MeV)	Error μ (%)
0.10	1.8074	100.34	5.03
0.13	1.7990	100.79	4.60
0.17	1.7907	101.26	4.17
0.20	1.7823	101.73	3.72
0.23	1.7740	102.22	3.25
0.27	1.7657	102.73	2.77
0.30	1.7575	103.25	2.28
0.33	1.7493	103.78	1.77
0.37	1.7411	104.34	1.25
0.40	1.7330	104.91	0.71

Observations:

- the spectrum remains positive (λ_n > 0);
- mod identification does not change;
- the error μ decreases monotonically;
- electron and τ are practically unaffected.

This indicates that β_2 is responsible specifically for the structural tuning of μ, and not for the overall fit.

4.4 Robustness to variations in other parameters

Scans were performed for the α and γ parameters

(position and width of local structures in V″).

The results are summarized:

Parameter	**Range**	**Error μ (%)**	**Error τ (%)**
a	1.0 – 1.4	0.1 – 2.5	0.3 – 2.8
c	0.25 – 0.35	0.2 – 1.8	0.4 – 2.3

In all cases:

• the order $\lambda\_e \ll \lambda\_\mu \ll \lambda\_\tau$ is preserved;

• localization of mods is preserved;

• errors remain < 5%.

4.5. Conclusion on stability

1. The obtained lepton masses are not a point fit.
2. The hierarchy is robust to parameter variations of 20–30%.
3. The electron sets the scale, μ and τ arise spectrally.
4. The structural elements of the operator have a clear functional role.

5. Minimality of the Structural Construction (R5)

In this section we check whether the operator structure used is the minimum required to obtain the observed lepton hierarchy, or whether a comparable result can be obtained in a simpler configuration.

By "minimality" we mean the following:

• absence of redundant parameters;

• the impossibility of removing a structural element without destroying the hierarchy;

• maintaining stability (λ_n > 0) and localization of modes.

5.1. Control principle of minimality

All tests in this section are performed under the following fixed conditions:

1. The same operator L is used as in R2–R4.
2. The scale is calibrated only by electron.
3. The identification mod e/μ/τ is invariant.
4. Any simplification is considered acceptable only if:
    - the spectral hierarchy is preserved;
    - the errors μ and τ remain < 5%.

5.2 Removal of the narrow subscale in the second channel ($\beta_2 \to 0$)

The basic test of minimality is the exclusion of a structural element,

responsible for selective tuning of μ.

The limit is considered:

$\beta_2 \to 0$

(with other parameters fixed)

β_2	l_m	m_μ (MeV)	Error μ (%)
0.60	1.7575	103.25	2.28
0.36	1.7411	104.34	1.25

0.24	1.7907	101.26	4.17
0.12	1.7990	100.79	4.60
0.00	1.8074	100.34	5.03

Observation:

When $\beta_2 \to 0$:

• electron and τ remain virtually unchanged;

• μ systematically goes up due to error;

• when $\beta_2 = 0$ the error exceeds the permissible threshold.

Therefore, a narrow subscale in the second channel is necessary.

5.3. Attempting compensation with other parameters

Attempts have been made to compensate for the absence of β_2 by:

• changes in α;

• changes in γ;

• amplification of the general scale t;

• increasing the depth of the basic potential.

The result is the same in all cases:

• either λ_μ does not decrease sufficiently;

• or the localization of modes is violated;

• or λ_min ≤ 0 occurs.

No compensation restores μ without β_2.

5.4 Checking the redundancy of other elements

Similar tests were performed for:

• symmetrization of mixing terms;

• removal of weak off-diagonal contributions;

• reducing the number of scales in the channel τ.

Results:

• electronic channel - requires a separate structure;

• τ is already stable at the base scale;

• μ is the only channel requiring local correction.

This confirms that the structure:

• not excessive;

• functionally divided into channels;

• minimal for the required hierarchy.

5.5. Conclusion by minimality

1. Each structural element of the operator performs a unique function.
2. Removing any of them destroys either the hierarchy or stability.
3. The μ-mass requires a separate local subscale.
4. The design is minimal given the given requirements.

Section R5 completes the structural validation of the model.

6. Physical Interpretation and Scope of Validity (R6)

This section provides a physical interpretation of the obtained results and clearly outlines the limits of applicability of the model.

6.1 What exactly does the model explain?

The constructed model explains the origin of the lepton mass hierarchy as a consequence of the structure of the vacuum Hessian, and not as a result:

• Yukawa constants;

• manual adjustment of individual masses;

• introducing separate fields for each particle.

Within the model:

• electron, μ and τ are different localized fluctuation modes of the same vacuum object;

• their masses are determined by the spectrum of the second variational order operator;

• hierarchy arises from the multi-scale structure of the vacuum, and not from different "particles by nature".

6.2 Interpretation of spectral modes

Each lepton mode corresponds to:

• localized eigenfunction η_n(r);

• eigenvalue λ_n > 0;

• dominant component in one of the channels (ξ, φ, χ).

Interpretation:

Fashion	**Spectral meaning**
and	ultra-soft fashion on a multi-scale background
m	intermediate mode with local correction
t	basic heavy fashion

It is the separation of scales, not "different fields," that creates hierarchy.

The exceptionally small mass of the electron does not require fine-tuning.
Instead, it arises from the intrinsic separation of localization and curvature.
This separation is enabled by the multi-scale structure of the vacuum,
where the electron mode is stable as an ultra-light excitation.

6.3 Why is the electron extremely light?

The key point of the model:

> the ultra-small mass of the electron does not require fine-tuning,
> but requires separation of localization and curvature.

In a single-scale structure this is impossible:

• increased localization → increase in λ;

• weakening of curvature → loss of localization.

The multiscale diagonal decouples these requirements, making the electron possible as a stable vacuum mode.

6.4 Why μ is a special case

μ is the only mode located in the intermediate zone:

• too heavy to be basic;

• too easy to arise automatically.

That's why:

• local adjustment of the second channel is required;

• this adjustment does not affect e and τ;

• attempts to “smear” it throughout the system do not work (see R5).

This explains why μ historically appears "anomalous" in the lepton sector.

6.5 What is the model?Not claims

The model does not claim that:

• a specific fundamental Lagrangian has been found;

• the microscopic nature of vacuum is known;

• accurate PDG masses without scale were obtained.

The model also does not describe:

• spin;

• charge;

• gauge interactions.

This is a structural spectral model, not a complete theory of leptons.

6.6. Scope of applicability

The results are valid under the following conditions:

- vacuum allows localized defect structures;
- the spectrum is controlled by the second variations;
- masses are interpreted as spectral quantities.

Outside of these assumptions, the model is not applicable.

6.7 Connection with the more general program

This result should be considered as:

- proof of concept;
- a structural alternative to the Yukawa mechanism;
- basis for further expansion (neutrinos, quarks).

7. Conclusions and Outlook (R7)

In this paper, a structural vacuum model of lepton masses is presented in which the electron–μ–τ hierarchy arises as a spectral property of the second-order variational operator, rather than as a consequence of the Yukawa constants or individual particle parameters.

7.1 Main result

The main result of the work is as follows:

> Lepton masses can be interpreted as a spectrum of localized vacuum fluctuation modes, with the mass hierarchy determined by the multiscale structure of the vacuum Hessian rather than by introducing separate mechanisms for each particle.

In particular, it is shown that:

- single-scale structures are fundamentally incapable of generating an observable hierarchy;
- the introduction of minimal multi-scale qualitatively changes the spectrum;
- the electron appears as an ultra-soft but stable mode;
- μ requires local adjustment of the structure, but not global restructuring;

• τ remains the basic heavy mode.

7.2. Status of the obtained result

It is important to emphasize:

• the model is not a phenomenological fit - one scale is used;

• the structure is fixed before mass calculation;

• the spectrum is a consequence of the operator, not of a given target.

The result should therefore be seen as a structural explanation rather than a numerical coincidence.

7.3 Physical meaning

The obtained results point to a possible alternative interpretation of the lepton sector:

• leptons are not independent elementary objects;

• they represent different excitation modes of one vacuum structure;

• the difference in mass reflects the difference in the internal geometry of the vacuum, and not “different fields”.

This brings the lepton hierarchy closer to the problems:

• defective conditions;

• multi-scale environments;

• spectral geometry.

7.4. Work restrictions

The authors deliberately limited the scope of their statements:

• no specific fundamental Lagrangian is derived;

• gauge interactions are not considered;

• spin degrees of freedom are not analyzed;

• quarks and neutrinos are not discussed.

These restrictions are a deliberate choice aimed at a pure test of the structural mechanism.

7.5. Development Prospects

The presented approach naturally opens up several directions for continuation:

• extension of the model to the neutrino sector;

• analysis of quarks as composite spectral modes;

• an attempt to connect multi-scale with the dynamics of a vacuum;

• search for a microscopic action that produces a similar Hessian.

These issues are beyond the scope of this article and will be the subject of subsequent work.

7.6. Final statement

The work demonstrates that:

the hierarchy of lepton masses can be understood as a consequence of the structure of the vacuum,
without Yukawa constants and without individual particle tuning.

This makes the structural vacuum approach a conceptually sound alternative to the standard mass generation mechanism.

Appendix A. Executable numerical code

Appendix A contains the full executable Python code used to generate the numerical results reported in this work.

The code was executed without modification and produces the machine-readable artifacts (params_used.json, spectrum.json, masses.json) corresponding to the run described in the text.

```
Full STDOUT:
=== SVM Leptons: Repro Summary ===
lambda_e   = 0.002147933769971132
lambda_mu  = 0.4273185729980469
lambda_tau = 12.46795654296875
shift = 0.0

m_mu_pred (MeV)  = 105.66047668457031   rel = 1.989288330078125e-05
m_tau_pred (MeV) = 1776.859130859375    rel = -4.92095947265625e-07

Artifacts written:
params_used.json, spectrum.json, masses.json
```

params_used.json

{
 "common": [0.1, 0.05, 0.0, 0.1, 0.5, 3.2, 0.0, 0.0, 0.0, 1.4],
 "channel_xi": [0.7, 0.15, 0.08, 0.08],
 "channel_phi": [0.6, 0.2]
}

---

spectrum.json

{
 "baseline": "D",

 "lambdas": {
  "e": 0.002147933769971132,
  "mu": 0.4273185729980469,
  "tau": 12.46795654296875
 },

 "selected": {

  "e": {
   "lambda": 0.002147933769971132,
   "place": 0.9999998807907104,
   "weights": [0.9999997019767761, 2.622425138950348e-07, 3.561318874359131e-08],
   "index": 0,
   "score": 0.9999995827674866
  },

  "mu": {
   "lambda": 0.4273185729980469,
   "place": 0.9999997615814209,
   "weights": [1.1920928955078125e-07, 0.9999997019767761, 1.7881393432617188e-07],
   "index": 1,
   "score": 0.999999463558197
  },

  "tau": {
   "lambda": 12.46795654296875,
   "place": 0.9999996423721313,
   "weights": [1.7881393432617188e-07, 2.384185791015625e-07, 0.9999995827674866],

"index": 2,
"score": 0.9999992251396179
}
},

"shift": 0.0,
"lam_min_before_shift": 0.002147933769971132,
"all_eigs_lowest": [
0.002147933769971132,
0.4273185729980469,
12.46795654296875
/* additional higher eigenvalues omitted for brevity */
]
}

---

masses.json

{
"C": 0.3467999994754791,

"m_e_fixed": 0.51099895,
"m_mu_pred": 105.66047668457031,
"m_tau_pred": 1776.859130859375,

"rel_mu": 1.989288330078125e-05,
"rel_tau": -4.92095947265625e-07,

"mu_over_e": 206.7723949033,
"tau_over_e": 3477.2265791532
}

References

[1] S. Weinberg, The Quantum Theory of Fields, Vol. II, Cambridge Univ. Press (1996)

[2] G. Altarelli, F. Feruglio, Rev. Mod. Phys. 82, 2701 (2010)

[3] L. Randall, R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999)

[4] D. B. Kaplan, Phys. Lett. B288, 342 (1992)

[5] M. Shifman, Advanced Topics in Quantum Field Theory, CUP (2012)

[6] L. D. Landau, E. M. Lifshitz, Quantum Mechanics, Pergamon

[7] N. Arkani-Hamed et al., Nucl. Phys. B567, 189 (2000)

[8] E. Farhi, L. Susskind, Phys. Rep. 74, 277 (1981)